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Fixing the Dilaton with Asymptotically-Expensive Physics?

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Abstract

We propose a general mechanism for stabilizing the dilaton against runaway to weak coupling. The method is based on features of the effective superpotential which arise for supersymmetric gauge theories which are not asymptotically free. Consideration of the 2PI effective action for bilinear operators of matter and gauge superfields allows one to overcome the obstacles to constructing a nonvanishing superpotential.

1. Introduction

String theorists would love to calculate the mass of the electron from first principles. Indeed, a quantitative determination of the elementary-particle spectrum predicted for string models at the weak scale is a prerequisite for any convincing comparison between string theory and experiment. The main obstacle to making such a determination has been a lack of understanding of how supersymmetry is spontaneously broken in string models, since supersymmetry breaking ultimately controls the pattern of masses which would be observed at experimentally-accessible energies.

Our present inability to understand supersymmetry breaking in string theory is not for want of trying. It was among the first problems addressed by early workers, who discovered the tantalizingly attractive mechanism for dynamical supersymmetry breaking [1] *via* gaugino condensation [2], [3]. Unfortunately, however promising its beginnings, this mechanism has proven to have some difficulties. One of these is its prediction of a superpotential which is exponential in the dilaton supermultiplet, $W = A\, e^{-a\, S}$, with a a positive constant. The resulting scalar potential is minimized for Re $S \to \infty$, in which limit the gauge coupling vanishes and supersymmetry does not break.

Generically, the scalar potential may have other minima in addition to the one at infinity. This happens, for example, when gaugino condensation occurs for a gauge group consisting of several factors [4], in which case the resulting superpotential has the form of a sum of exponentials:

$$W = \sum_n A_n\, e^{-a_n\, S}. \tag{1}$$

The problem with this scenario is that it is difficult to understand why the universe should not end up within the basin of attraction of the 'runaway' solution [5].

It has been argued that the 'runaway dilaton' problem is generic in string theory [6] . This is because the *v.e.v.*s of the dilaton multiplet's complex scalars determine the coupling constant, g, and vacuum angle, θ, of the low-energy gauge interactions according to the relation $\langle S \rangle = \frac{1}{g^2} + \frac{i\theta}{8\pi^2}$. Since flat space is known to solve the field equations for noninteracting strings — *i.e.* for $g = 0$ — the scalar potential might be expected to always

admit a minimum when Re $S \to \infty$.

The purpose of this letter is to point out some loopholes in the argument that the superpotential must vanish for large S. We do so by constructing some simple models which do not have this property. One class of models to which we are led involves low-energy gauge groups which are *not* asymptotically free. We therefore devote some discussion to special issues which arise when using the effective superpotentials for nonasymptotically-free effective gauge theories.

We start by re-examining the superpotential, eq. (1), arising from traditional gaugino condensation. Our main observation is that the problem of runaway solutions only arises if all of the exponents in this equation are negative. For instance, in global supersymmetry the scalar potential is minimized by the extrema of the superpotential, so if both a_1 and a_2 are positive, then:

$$W = A_1\, e^{-a_1\, S} + A_2\, e^{+a_2\, S}, \tag{2}$$

has only the single vacuum solution:[1] $\overline{S} = \ln\left(a_1 A_1 / a_2 A_2\right)/(a_1 + a_2)$.

Once coupled to supergravity this solution can also break supersymmetry, depending on the form taken by the Kähler potential, K. For example, the usual perturbative string theory result [8]: $K = -\ln(S + S^*) - 3\ln(T + T^*)$ — where T denotes the complex scalar containing the 'breathing' modulus — leads to the 'no-scale' [9] scalar potential $V = |D_S W|^2 (S + S^*)/(T + T^*)^3$. Here $D_i W \equiv \partial_i W + \partial_i K\; W$ (with $i = S, T$) are order parameters for supersymmetry breaking. V is minimized by ensuring $D_S W = 0$, for any T. So long as other contributions to the potential do not drive $T + T^* \to \infty$, $D_T W \neq 0$ at the minimum of V, and so supersymmetry is broken. A similar thing happens if the theory is required to be invariant under T duality. In this case W depends nontrivially on T, although T-duality invariance nonetheless ensures that $|D_S W|^2 = 0$ but $|D_T W|^2 \neq 0$ at the minimum, breaking supersymmetry [10].

Of course, if the supersymmetry-breaking scale in the above scenario is taken to be much smaller than the Planck mass — which is taken here as unity — then some fine-

[1] Dilaton potentials blowing up at infinity were discussed in the context of S duality in [7].

tuning is required of the parameters A_i or a_i. Here we do not pursue the extent to which this fine-tuning may be ameliorated, but focus instead on the runaway-dilaton problem, and how a superpotential like eq. (2) might be generated in the first place.

2. Non-Asymptotically Free Models

Some intuition as to how to generate a superpotential like eq. (2), which blows up for small couplings, can be obtained as follows. Consider a quartic potential for a real scalar field: $V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}g^2\,\phi^4$. When evaluated at its minimum, $\overline{\phi} = \pm m/g$, the scalar potential becomes $V(\overline{\phi}) = -m^4/4g^2$, a result which blows up as $g \to 0$. The potential, once minimized, is singular in the weak-coupling limit because g premultiplies the highest power of ϕ in $V(\phi)$, and so any nonzero value for g, no matter how small, qualitatively alters the field configuration which minimizes V. Operators, such as $\frac{1}{4}g^2\,\phi^4$, whose couplings can appear singularly in this way are well known in renormalization-group applications within condensed-matter physics, where they can play important roles in analyses even if they are nominally irrelevant in the RG sense. They are known there as 'dangerous irrelevant operators' [13]. This suggests that an exponential of $+S$ might be achieved by introducing a field whose highest-dimension term in W arises premultiplied by a positive power of e^{-S}.

Another useful piece of intuition comes from the observation that strongly-coupled supersymmetric gauge theories tend to produce superpotentials which are proportional to the appropriate power of the renormalization-group invariant scale: $W \propto \Lambda^3$, where for pure gauge theories $\Lambda \propto M_s e^{-8\pi^2/bg^2}$, where M_s is a high-energy cutoff (the string scale, say) and $b/16\pi^2$ is the coefficient of the one-loop beta function.[2] Since it is the gauge coupling at the string scale which is related to the dilaton by, $1/g^2(M_s) \propto \mathrm{Re}\ S$, the negative argument of the exponential is related to the sign of the coefficient, b. This suggests[3] the potential utility of working with nonasymptotically free (NAF) gauge theories.

Based on these observations, we attempt to obtain eq. (2) by constructing a model for which the gauge group consists of two factors, $G = G_1 \times G_2$, with G_1 asymptotically free

[2] Our normalization of b here differs by a factor of $16\pi^2$ from our earlier conventions [11].

[3] Of course this argument is merely suggestive, since in the presence of matter it is the conformal anomaly coefficient, c, rather than the beta-function coefficient, b, which appears in the exponent with S in W [12].

and G_2 not. In this case, there is no value of $Re\, S$ for which *both* Λ_1 and $\Lambda_2 \to 0$, and so for which W might vanish. For simplicity, we take G_1 to consist of a gauge theory without matter, and focus in what follows on the physics associated with the second factor, G_2.[4]

To make this precise consider supersymmetric Quantum Chromodynamics (SQCD) having N_c colours and N_f flavours. The superfields of the theory consist of a left-chiral, left-handed-spinor gauge supermultiplet, $\mathcal{W}^a$ ($a = 1, \dots, N_c^2 - 1$), and left-chiral scalars representing the quarks and antiquarks: ${Q^i}_\alpha, {\overline{Q}_i}^\alpha$ ($i = 1, \dots, N_f$, $\alpha = 1, \dots, N_c$). In the absence of a superpotential the global internal symmetries of the model (modulo anomalies) are $G_f = SU_L(N_f) \times SU_R(N_f) \times U_B(1) \times U_A(1) \times U_R(1)$, with respect to which the fields are assigned the following transformation rule:

$$\mathcal{W}^a \sim \Bigl(\mathbf{1}, \mathbf{1}, 0, 0, \frac{3}{2}\Bigr), \qquad {Q^i}_\alpha \sim \Bigl(\mathbf{N_f}, \mathbf{1}, 1, 1, 1\Bigr), \qquad {\overline{Q}_i}^\alpha \sim \Bigl(\mathbf{1}, \overline{\mathbf{N}}_\mathbf{f}, -1, 1, 1\Bigr). \tag{3}$$

The first two numbers in these expressions denote the representation of $SU_L(N_f) \times SU_R(N_f)$ under which the field transforms, while the last three numbers give their charges under the three $U(1)$ generators, B, A and R.

For SQCD the beta function and scale anomaly coefficients are $b = 3N_c - N_f$ and $c = N_c - N_f$. For $N_f < 3N_c$ this theory is asymptotically free. The possible nonperturbative superpotentials and general structure of the theory have been extensively investigated for any value of N_f and N_c [15].[5]

Based on the $\frac{1}{4} g^2 \phi^4$ analogy presented above, we supplement SQCD with an additional colour-singlet left-chiral scalar superfield, ${\mu^i}_j$, which transforms in the same way as would a quark mass term: ${\mu^i}_j \sim \Bigl(\overline{\mathbf{N}}_\mathbf{f}, \mathbf{N_f}, 0, -2, 1\Bigr)$. We take the superpotential in the Higgs phase to be [18]

$$W = \mathrm{Tr}\Bigl(\mu Q \overline{Q}\Bigr) + \frac{\lambda}{3}\, \mathrm{Tr}\Bigl(\mu^3\Bigr). \tag{4}$$

where λ is a Yukawa coupling which explicitly breaks the axial symmetry, $U_A(1)$, but none of the other symmetries. In fact a cubic superpotential in μ is the only possible form which

4 In general we will not need to have two gauge groups to stabilize the dilaton, we may in principle use corrections to the Kähler potential together with one single exponential [14].

5 For recent reviews see refs. [16] and [17].

does not explicitly break the R-symmetry. Furthermore, we imagine λ to be small enough to justify ignoring this coupling in comparison with the gauge couplings when determining the vacuum structure of the model.

Since our goal is to determine the potential which fixes the *v.e.v.* of the dilaton field, S, we now construct the effective action, Γ, which generates the model's irreducible correlation functions.[6] We take the arguments of Γ to be S and $\mu^i{}_j$, as well as the expectation values of the gauge-invariant fields $M^i{}_j \equiv \left\langle Q^i{}_\alpha \, \overline{Q}_j{}^\alpha \right\rangle$ and $U \equiv \langle \mathcal{W}^a \, \mathcal{W}_a \rangle$. (For $N_c < N_f$ we imagine the expectation value of the baryon operator, $B^{i_1 \cdots i_{N_c}} = \left\langle \epsilon^{\alpha_1 \cdots \alpha_{N_c}} \, Q^{i_1}{}_{\alpha_1} \cdots Q^{i_{N_c}}{}_{\alpha_{N_c}} \right\rangle$ to be zero.)

A great deal is known about the exact superpotential which appears in $\Gamma[U, M, \mu, S]$. The non-perturbative terms are required on general grounds to be linear in both S and $\mu^i{}_j$ [11]. Furthermore, its dependence on U and $M^i{}_j$ is completely determined by the gauge and global symmetries of the problem [15], together with the anomaly-cancelling transformation rules for S: $e^{-8\pi^2 S} \sim \Big(\mathbf{1}, \mathbf{1}, 0, 2N_f, 3N_c - N_f\Big)$. After eliminating $U = U(M, \mu, S)$ by extremizing Γ with respect to U, the result becomes:

$$W(M, \mu, S) = \mathrm{Tr}\Big(\mu M\Big) + \frac{\lambda}{3} \, \mathrm{Tr}\Big(\mu^3\Big) + k \, \left(\frac{e^{-8\pi^2 S}}{\det M}\right)^{1/(N_c - N_f)} , \tag{5}$$

where $k = N_c - N_f$. Extremizing with respect to $M^i{}_j$, and substituting the result back into Γ then gives the superpotential

$$W(\mu, S) = \frac{\lambda}{3} \, \mathrm{Tr}\Big(\mu^3\Big) + k' \Big(e^{-8\pi^2 S} \; \det \mu \Big)^{1/N_c} , \tag{6}$$

where $k' = N_c$. If $\mu^i{}_j$ were a constant mass matrix, eq. (6) would give the superpotential for S in SQCD. It is noteworthy that, so long as $k' \neq 0$ and μ is held fixed, the result has runaway behaviour to $S \to \infty$ *regardless* of the values of N_c and N_f.

The final step is now to extremize W with respect to the field $\mu^i{}_j$, to obtain the overall superpotential for S. The extremum is obtained for $\mu^i{}_j = \Big(-\lambda \, e^{8\pi^2 S/N_c}\Big)^{N_c/(N_f - 3N_c)} \; \delta^i{}_j$,

[6] Notice that Γ is *not* the Wilson action of the theory. It is therefore potentially subject to holomorphy anomalies, which we believe to play no role here, in the presence of massless particles [19].

and using this in eq. (6) gives:

$$W(S) = k'' \Big(\lambda^{N_f} \, e^{24\pi^2 S} \Big)^{1/(N_f - 3N_c)}, \tag{7}$$

with $k'' = (-1)^{3N_c/(N_f - 3N_c)} \, (N_f - 3N_c)/3$. Notice that eq. (7) takes the simple form $W \propto \Lambda^3$ when expressed in terms of the renormalization group invariant scale, $\Lambda \propto e^{-8\pi^2 S/b}$. Eq. (7) gives the desired positive exponential of S, but only if $N_f > 3N_c$, and only if the theory is not asymptotically free. When this is combined with the potential for another, asymptotically-free, factor of the gauge group we obtain a superpotential of the form of eq. (2).

Several comments bear emphasis at this point. Notice first that the singular dependence implied by eq. (7) for W in the limit $S \to \infty$ is very similar to the singularity of $V(\overline{\phi})$ in $\frac{1}{4} g^2 \, \phi^4$ theory as $g \to 0$. This is most easily seen by examining eq. (6) for the special case where ${\mu^i}_j$ is proportional to the unit matrix, ${\mu^i}_j \equiv \mu \, {\delta^i}_j$. In this case the first term of (6) is cubic in μ, while the second involves μ raised to the N_f/N_c power. The second term is therefore 'dangerous' — *i.e.* involves the higher power of μ — precisely when $N_f > 3N_c$. Since a positive power of $e^{-8\pi^2 S}$ premultiplies this second term, this is also when W is singular as $S \to \infty$.

The possibility of having potentials for S which diverge as $S \to \infty$ is reminiscent of what was found years ago for the T field. In this case, even though large T corresponds to weak worldsheet coupling, standard T-dual potentials blow up as $\mathrm{Re}\, T \to \infty$ [10]. The underlying reasons for these divergent behaviours appears to differ in detail, however, since the large-T singularity can be attributed to a large value of the 10D string coupling, g_{10}, even when the 4-D coupling, g_4, is weak. These limits are mutually consistent for large $\mathrm{Re}\, T$ (or, equivalently, large compactification radius, R) because g_{10} and g_4 are related by $1/g_4^2 = R^6/g_{10}^2$.

Finally, since ${\mu^i}_j$ plays the role of a quark mass matrix, its eigenvalues must be smaller than the scale Λ if there is to be a range of scales for which the theory is to be weakly coupled and not asymptotically free. We may always ensure this to be true by taking the Yukawa coupling, λ, to be sufficiently small. When this is done, the degrees of freedom

in the energy range $\mu < E < E_{\rm max}$, where $E_{\rm max} = \Lambda$, or some other scale at which new degrees of freedom become important, describe a non-asymptotically-free theory of supersymmetric quarks and gluons. For $E < \mu$ the quarks and their superpartners may be integrated out, leaving an asymptotically-free model at these lower scales.

3. Discussion

We see that both signs are possible for the arguments of the exponentials which appear in the superpotential for S, depending on the matter content of the strong-coupling physics which generates it. In particular, the desired positive exponentials of S can be generated by non-asymptotically free SQCD-like gauge theories.

There are a number of criticisms which might be raised against using nonasymptotically-free gauge theories in the way we have. We now address some of these.

• *(1):* Perhaps the easiest objection to deal with is the widely held belief that only asymptotically-free gauge theories can be consistently defined as interacting quantum field theories. Even if this proves to be true, asymptotic freedom is not a fundamental requirement for an *effective* theory, which only describes the degrees of freedom at very low energies (or very long distances). Quantum Electrodynamics is probably the most famous example of such an effective theory, which is now believed to be the low-energy approximation to the more complete Standard Model of electroweak interactions.

This point of view is all the more inevitable within the context of string theory, where the entire discussion of four-dimensional supersymmetric theories is intended as a low-energy description of a more fundamental string theory. Furthermore, many four-dimensional string compactifications are known which actually have low-energy spectra giving rise to nonasymptotically-free gauge interactions within the effective low-energy theory [10].

• *(2):* A potentially more compelling objection argues that, although an effective superpotential like eq. (5) can arise in SQCD for sufficiently small N_f, it does *not* arise when $N_f > 3N_c$. This line of argument proceeds in one of two ways.

One form of this objection argues that the quantity $\det M$ itself vanishes for $N_c < N_f$ because the same is true for $\det\left(Q\overline{Q}\right)$ [16], [17].[7] The weakness in this argument lies in its making an insufficient distinction between the effective action and the Wilson action.[8] The Wilson action, S_W, describes the dynamics of the low-energy degrees of freedom of a given system, and is used in the path integral over these degrees of freedom in precisely the same way as is the classical action. The Wilson action for SQCD at scales for which quarks and gluons are the relevant degrees of freedom would therefore depend on the fields $\mathcal{W}^a$, ${Q^i}_\alpha$ and ${\overline{Q}_i}^\alpha$. As a result, the vanishing of $\det(Q\overline{Q})$ would indeed preclude the generation of a superpotential of the type $\left[e^{-8\pi^2 S}/\det(Q\overline{Q})\right]^{1/(N_c-N_f)}$ within the Wilson action.

By contrast, it is the effective action, Γ, which is of interest when computing the *v.e.v.*s of various fields. And it is ${M^i}_j = \left\langle {Q^i}_\alpha {\overline{Q}_j}^\alpha \right\rangle$ which appears as an argument of Γ. Since the expectation of a product of operators is not equal to the product of the expectations of each operator, it need not follow that $\det M = 0$ when $N_c < N_f$ [3].

• *(3):* It is equally clear that the failure of the fields ${M^i}_j$, $B^{i_1 \cdots i_{N_c}}$ and $\overline{B}_{i_1 \cdots i_{N_c}}$ to satisfy the 't Hooft anomaly-matching conditions [16] does not argue against the superpotential (5). Anomaly matching says that the physical degrees of freedom at any scale must have anomalies which reproduce the anomaly of the underlying degrees of freedom. This is a constraint on which fields can appear in the path integral over the Wilson action at any scale, since it is the functional measure for these fields which gives the anomaly. But it is *not* a constraint on the 2PI action which we are considering since the arguments of this action are not quantum fields to be integrated over, and so do not have anomalies. Anomalies appear in the 2PI action simply as terms which explicitly break the corresponding symmetry, and this has been used to construct the superpotential of (5).

• *(4):* An alternative objection concedes the necessity of using the effective action, Γ, rather than the Wilson action, S_W. It also concedes the consistency of eq. (5) with all of the symmetries of the theory. The objection is that the constant k, which premultiplies the interesting term in this equation, must equal zero.

[7] $\det(Q\overline{Q})=0$ because ${Q^i}_\alpha {\overline{Q}_j}^\alpha$, being a sum of dyadics, always has zero eigenvalues so long as $N_c<N_f$.

[8] See refs. [11] and [20] for a detailed discussion of this distinction within the context of gaugino condensation.

The vanishing of k might be argued as follows. For asymptotically-free theories the last term of eq. (5) is interpreted as being the result of gaugino condensation. This may be seen explicitly by recognizing that the gaugino condensate is given by $U = \partial W/\partial S$, with W given by eq. (5), (6) or (7). If the theory is not asymptotically free, the argument continues, then it is weakly coupled at low energies and so the low-energy dynamics cannot cause the gauginos to condense (or have any other strong-coupling consequences).

One difficulty with this argument is that it assumes it is the low-energy dynamics which must be responsible for the nonperturbative terms in the effective superpotential. In a theory which is not asymptotically free, it is the higher energy degrees of freedom which are more strongly coupled than the lower energy ones. As a result one would expect a term of the form of eq. (6) to be directly generated in the Wilson action at high energies. This term can then survive down to low energies to contribute to Γ. A second, perhaps more convincing, difficulty with this argument is given in the next item.

• *(5):* A variation of the previous point argues that instanton calculations do not produce such a nonvanishing superpotential [17]. However, instanton calculations do not always explicitly produce nonzero contributions to quantities which are known not to vanish. For instance, instantons give a vanishing gaugino bilinear, $\langle \overline{\lambda} \lambda \rangle$, because there are not enough fermion fields to soak up all of the zero modes which arise in the instanton path integral. We nevertheless know that the expectation value of the gaugino bilinear does not vanish since it can be determined by factorizing the expectation values of correlations of more powers of gaugino fields, for which instanton contributions are nonzero [3]. Therefore, we regard the absence of instanton contributions to be insufficient evidence for a vanishing superpotential.

Perhaps the best argument against this, and the previous, point is based on continuity with the case $N_c \geq N_f$ [3]. A powerful technique for determining the constants, k, k' and k'' in the superpotentials is to give large masses to some of the quarks, and then to integrate these out. Once this is done one must obtain the correct result for the theory having fewer quarks. This permits relations to be derived between theories having the same value of N_c but differing values of N_f. If it is supposed that no nonperturbative superpotential

can appear for $N_f > 3N_c$, then it is difficult to obtain the known superpotential in the asymptotically-free case where N_f is smaller. The same objection does not apply if eq. (5) applies for *all* values of N_c and N_f.

• *(6):* With our final point we address a minor puzzle. According to Seiberg, SQCD with $N_f > N_c + 1$ is dual to a supersymmetric model having N_f flavours and $\widetilde{N}_c \equiv N_f - N_c$ colours, coupled to an extra gauge-singlet field, $\widetilde{M}^i{}_j$, which has the quantum numbers of a mass matrix for the dual quarks, q^i. Since the condition for asymptotic freedom is $N_f < 3N_c$ for the original theory, and $N_f > 3N_c/2$ for its dual, it is impossible that *both* the original theory and its dual are not asymptotically free. The puzzle is this: for $N_f > 3N_c$ the dual theory is asymptotically free and so should dynamically generate a superpotential which vanishes in the weak-coupling limit. How can this share the same vacuum structure as the model constructed using the original variables?

To see that the dual theory indeed implies a superpotential which vanishes at weak coupling, even though it also involves the new gauge-singlet field, $M^i{}_j$, which is otherwise similar to $\mu^i{}_j$, proceed as follows. Following Seiberg we write the dual superpotential, including a quark mass matrix $\mu^i{}_j$ as:

$$\widetilde{W}\left(\widetilde{M}, \mu, \Omega, \widetilde{S}\right) = \rho \,\mathrm{tr}\, \mu \widetilde{M} + \mathrm{tr}\, \Omega \widetilde{M} + \frac{\lambda}{3} \mathrm{tr}\, \mu^3 - N_c \left(\det \Omega \; e^{8\pi^2 \widetilde{S}}\right)^{1/N_c}, \tag{8}$$

where ρ is an undetermined constant scale, Here $\Omega^i{}_j \equiv \left\langle q^i \overline{q}_j \right\rangle$, and we have added the same cubic term in μ as used in the model of the previous sections. Extremizing this superpotential with respect to $\widetilde{M}$ implies $\Omega^i{}_j = -\rho \mu^i{}_j$, and solving the field equation for $\mu^i{}_j$ in the result gives $\mu^i{}_j = \left((-)^{N_f} \lambda^{N_c} (\rho)^{-N_f} e^{-8\pi^2 \widetilde{S}}\right)^{1/(N_f - 3N_c)} \delta^i_j$. This gives the dual superpotential:

$$\widetilde{W}(\widetilde{S}) = \tilde{k} \left(\left(\lambda \rho^{-3}\right)^{N_f} e^{-24\pi^2 \widetilde{S}}\right)^{1/(N_f - 3N_c)}, \tag{9}$$

where $\tilde{k} = (-)^{-3N_f/(N_f - 3N_c)} \left(N_f - 3N_c\right)/3$. This gives $\widetilde{W}$ proportional to a negative exponential of $\widetilde{S}$, as claimed. The problem is to see the consistency of this result with W being proportional to a positive exponential of S.

The resolution to this puzzle hinges on the connection between the gauge couplings in the two models. This is given by the duality relation for the RG invariant scales of the two theories, which was proposed for the case $N_f < 3N_c$ in ref. [15], and which we extend here to general N_f and N_c. Equating the *v.e.v.* found for ${\mu^i}_j(\widetilde{S})$ in the dual theory with that found for ${\mu^i}_j(S)$ in the original model implies the relation:

$$\Lambda^{3N_c-N_f}\widetilde{\Lambda}^{2N_f-3N_c} = (-)^{N_f-N_c}\,\rho^{N_f}. \tag{10}$$

This duality relation in particular implies that Re $\widetilde{S} \to -\infty$ when Re $S \to +\infty$. Since $\widetilde{W}$ involves positive powers of $e^{-\widetilde{S}}$, this is consistent with our finding that W involves positive powers of e^{+S}.

We conclude, therefore, that string theory may well solve its runaway dilaton problem by producing a low energy spectrum which is described by a nonasymptotically-free effective theory. If so, then run-of-the-mill mechanisms for supersymmetry breaking — like gaugino condensation — can generate superpotentials for which the dilaton *v.e.v.* is fixed and cannot run off to weak coupling ($S \to \infty$). Thus, nonasymptotically free theories bear more detailed study to see whether this kind of mechanism can avoid some of the other problems (such as fine-tuning or cosmological constant problems) to which string theories seem prone. It would be interesting to find a connection between the field theoretical ideas discussed in this paper and the recent developments understanding $N = 1$ supersymmetric theories from the D-branes structure of M-theory [22].

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